Theoretical Rook Endgames

By

Sam Shankland



Contents

Key to Symbols used Foreword by GM Karsten Müller Preface		4 5 7			
			Introduction		11
1	Lucena Position	13			
2	One Pawn, Basic Knowledge	23			
3	Vancura Defense	63			
4	Vancura Transitions	79			
5	Side Cut	95			
6	Lone Doubled Pawns	103			
7	Lone Connected Pawns	113			
8	Lone Split Passed Pawns	137			
9	Two Pawns Against One on the Same Side	159			
10	Three Pawns Against Two on the Same Side	177			
11	Four Pawns Against Three on the Same Side	197			
12	Two Connected Passed Pawns Against One Outside Passed Pawn	235			
13	Pawn on the Seventh Rank with Rook on the Eighth	251			
14	Rook in Front of the Passed Pawn	271			
15	Rook to the Side of the Passed Pawn	295			
16	Rook Behind the Passed Pawn	327			
17	Pawns on Both Sides with No Passed Pawns	347			
18	Rook Against Pawn(s)	359			
19	Pawnless Rook Endgames	379			
20	The Bare Bones	405			
Name Index		445			

Foreword by Karsten Müller

Endgame theory can be fun

Rook endings are the type of theoretical endgame which occur most often by far. They are worth studying as there are many positions which occur over the board regularly. Philidor's draw, Lucena's win and Vancura's draw are just the three most prominent examples. So there is already a vast literature on the subject. Why add two more books to the collection?

Rook endgames can be regarded as having two aspects. One theoretical and one practical. So Quality Chess decided to cover each aspect in a separate book. Sam Shankland has a systematic style as he has proven in his excellent books on pawn play (*Small Steps to Giant Improvement* and *Small Steps 2 Success*). So, he was a great choice for the theoretical work, the aptly named *Theoretical Rook Endgames*. Equally, Jacob Aagaard's creative genius, as demonstrated most recently in his *A Matter of Endgame Technique*, makes him an ideal choice for the sister work on practical rook endings, *Conceptual Rook Endgames*. Whilst the study of each book will be equally valuable to the practical player, it would be preferable for the reader to start with obtaining a full understanding of the theoretical aspects of rook endgames from Sam's book. These provide the fundamental building blocks to the practical aspects of such endings as demonstrated in Jacob's book, which provides more advanced material.

In *Theoretical Rook Endgames*, Sam Shankland presents the basic theory of rook endgames with the most important positions and guidelines. The book is well-structured, and the presentation is logical and systematic. Well-chosen terms are used for the motifs. The book is particularly good at sharpening the intuition by setting out clear guidelines and the exceptions which apply to those guidelines. He also deals with two types of endings which often arise from rook endings, being rook against pawn and rook against knight (as well as other pawnless rook endgames). This shows that Sam's work is well grounded.

I particularly like his idea to present winning pawn structures, together with the plans and conditions which relate to each structure. They are well worth memorizing, as endgames such as those with four against three pawns on one wing, or three against three pawns on one wing and an extra attacking passed pawn, occur frequently and are not easy to handle over the board.

Of course, I already knew most of the material, but Sam's instructive presentation was a joy to follow and sometimes he also breaks new ground. Examples include his presentation of rook and doubled pawns against rook, and his deep discussion of triangle constellations with rook and two pawns against rook and pawn.

Sam has also added many recent games of his own. These prove both that such theoretical rook endgames often occur in practice, and that they are difficult to successfully navigate over the board.

I hope that Sam's work will give you as much pleasure as it has given me.

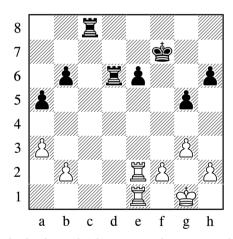
GM Dr Karsten Müller Hamburg, April 2023

Preface

I never had much formal training when I was young. I was born and raised in the United States, which did not have the chess culture of Europe and Asia at the time. I went through the full American K-12 education system, and I was not able to dedicate myself full time to chess until I was an adult. I was largely self-taught, and while I had some early coaches once per week for whom I am immensely grateful, I was never exposed to any Soviet-style endgame training. My rook endgames were preposterously bad, as the following disaster clearly illustrates.

Sam Shankland - Alex Lenderman

Saint Louis 2010



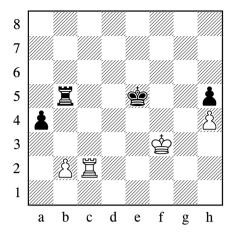
This position is obviously dead equal. There was a thirty-move draw rule in effect, so when I made my next move, I offered a draw and was surprised that the game did not end immediately.

31.曾g2 罩c4 32.罩e4?!

This was hardly a losing blunder, but the first step in the wrong direction. The position becomes a little unpleasant once the rook reaches b3.

Any normal move would be fine. 32.\mathbb{Z}e3 for example.

32... \Begin{aligned} 33. \Begin{aligned} \Beg



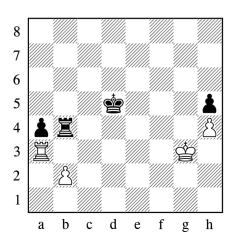
Up to here, my play had been somewhat lame, but the position is still well within the realm of being drawn. My machine gives 0.00 with any remotely reasonable-looking move.

There is no universe where this rook should have ever left c2. It was well placed for maximum checking distance and it defended the b2-pawn. Still, the machine insists on dead equality.

46... \$\d5 47.\alpha a3?

But not after I compound my error after putting my rook on a ridiculous square!

47...≌b4! 48.фg3



The next thing I know, both of my pieces are stuck and passive. Alex did not do anything special – I did it all to myself. White is now lost, though luckily I was given another chance.

48...**⊈c6**?

48... 堂e4! White is in zugzwang and very lost. 49. 置a2 堂d3 The king comes to c2 and that would be that.

49.\c3†!

I will give myself one consolation exclamation mark for undoing the ridiculous decision I made a couple moves before.

Looking back on this game, I think Alex did me a big disservice by not taking his chance and beating me. It would have been much harder to remain in denial about my rook endgame ineptitude if I had been punished for taking decisions that violate every rule of chess understanding.

I was over 2500 when I played this game. I have come a long way since then, but my lack of proper endgame training when I was young always dragged me down. I left two half points on the table in the 2018 Olympiad as a 2700+ player, both in rook endgames. Not only did this cost me ten rating points, but it also probably cost the United States the Gold Medal. Both games are featured in this book.

My main ambitions in chess lie behind the board. I am a full-time player, and far from retirement age. I have won a lot of accolades and have already had a fantastic career. But to this point, I have topped out at number 22 in the world. My greatest ambition for the next ten years is to drop that number and rise even further in the world rankings.

Many of my peers have questioned my decision to dedicate as much time as I do to producing content rather than focus exclusively on my own training. Indeed, this is my fourth book for Quality Chess, and I have written nearly a dozen Chessable e-books!

The reason that this works for me, and the reason that I can keep a high level of play while also contributing to chess literature, is that I choose to take on projects that I believe will help me grow as a chess player. Every single word I have ever written for both publishers had the same purpose in mind: to make me a better chess player. I have chosen topics that I am interested in learning, and the great majority of the time I spend on writing a book is time studying chess and trying to grow my own knowledge and understanding.

While I have done a lot of writing before, this book was a different animal. Between *Small Steps to Giant Improvement, Small Steps 2 Success, Grandmaster Training Camp 1 – Calculation!*, and all my opening courses on Chessable, I'm guessing the most demanding project I ever took on lasted about two months from writing the first word and submitting the final draft for publication.

This one took three-and-a-half years.

When the COVID-19 pandemic hit in early 2020, nobody knew how long we would be in lockdown, and what the world would look like in a year, two years, or five. All I knew

was that for the foreseeable future, I would be stuck at home, unable to play over-the-board chess. But I also knew that someday over-the-board chess would return, and that I needed to keep up my training regime as best I could. I also knew I would need something to keep me busy. After my mistakes at the Olympiad, my trainer Jacob Aagaard thought I should study more rook endgames. We agreed that writing *Theoretical Rook Endgames* would feed two birds with one scone, keeping me busy for a long time while also improving a part of my game that clearly needed work.

This book might only be slightly more pages than any other work I have done, but it was oceans harder. I worked tirelessly for weeks at a time until I would burn out, take a break, and come back for more. In the 16 months between COVID hitting and my next overthe-board tournament, I only managed to get about 70% of the way through. Then when the tournament schedule picked up, my writing pace slowed to a crawl as I was unable to dedicate any serious time to it. But a sudden lack of events between October 2022 and June 2023 left me more time available to complete what I started.

Of all the work I have ever done in chess literature, video series, articles and content, this is the one I am the proudest of by a wide margin. It was a labor of love, a journey of growth and a source of massive frustration, all in one. It is my sincere hope that the years I spent on this work will help all dedicated readers improve their rook endgame play, and by extension, become stronger players.

Sam Shankland Walnut Creek, California July 2023

Chapter 6

Lone Doubled Pawns

Two pawns against zero tends to win in rook endgames, but there are some notable exceptions. The most obvious is the case of doubled pawns. For example, the following position is a clear draw.

8 7 6 5 4 3 2 1 a b c d e f g h

Example 1 – Pawns on e4 and e3

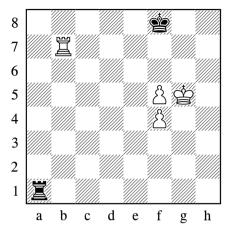
White's additional pawn on e3 doesn't help him in the slightest. Black has set up a blockade, and he can block two pawns on the same file just as easily as he can block one.

1.罩f5† 空e6 2.空f4 罩a1

White's king has nowhere to hide. It's a dead draw.

Fair enough, but this was a case where Black easily managed to set up a Philidor defense on the very first move, and when the pawns were still not far down the board. Unfortunately for the defending side, things get a lot worse when the pawns are further advanced and the Philidor defense is harder to come by.

Example 2 – Pawns on f5 and f4

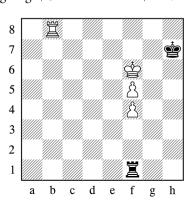


White's f4-pawn will obviously not become a queen if the f5-pawn does not promote first, and if the f5-pawn promotes, the game is already decided in White's favor. But the rear pawn can serve as a shield for White's king, which means Black must defend a bit more accurately than he would otherwise have to.

A mistake is:

1...∲g8?

If White's f4-pawn were not on the board, this move would not be the easiest route to a half point, but would still be sufficient, as we saw in the long and short side defense on page 29. As is, the extra pawn on f4 will provide White's king and the further advanced f-pawn with the shelter they need. 2. 全 g6 量 g1 † 3. 全 f6 量 f1 4. 国 b8 † 全 h7



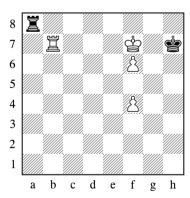
If White's pawn on f4 were removed from the board, his king would not be able to move to f7 or e7 without losing his last pawn, and \$\delta\$f6-e6 would allow ...\$\delta\$h7-g7. So, he would need to play a move like \$\bar{2}\$b8-f8 to overprotect the remaining f-pawn and prepare for \$\delta\$f6-e7. Then Black would save the game with ...\$\Bar{2}\$f1-a1 and giving lateral checks. Now, White can freely advance his king to the seventh rank without having to bother about bringing his rook to f8 first.

5.\\dagger{1}f7!

Here we see the point. Black cannot take on f5, and the f4-pawn slows him down just enough for White to win the game.

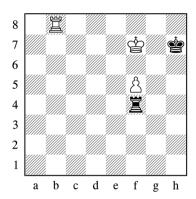
5...¤xf4

Transferring the rook to check from the side also doesn't help: 5...\(\mathbb{Z}\)a1 6.\(\mathbb{Z}\)b7 White is free to use his rook to secure the king on the seventh rank as it did not have to come to f8 first to help the king move. 6...\(\mathbb{Z}\)a2 (6...\(\mathbb{Z}\)f1 7.f6 \(\mathbb{Z}\)xf4 8.\(\mathbb{Z}\)e8\(\mathbb{Z}\) White is ready for f6-f7 next, and his rook can block any check on e4 by sliding to e7. Time for Black to resign.) 7.f6 \(\mathbb{Z}\)a8



Normally, even this position would be a draw if White did not have the f4-pawn. Now, White has a very instructive winning plan, highlighting its usefulness. 8.\$\dongo e^7\$ \$\dongo g6 9.\$f7 \$\dongo g7\$ Without the f4-pawn, White would be unable to break through here, as the rook cannot leave the seventh rank

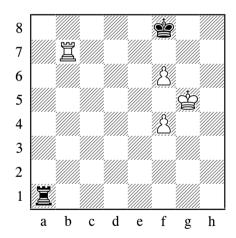
without allowing a check. But with the f4-pawn, the king can be booted off g7. 10.f5 and f5-f6† is coming, winning.



6.f6

White will reach the Lucena position, and win.

2.f6 \(\mathbb{Z}\)a1



Black has successfully implemented the Philidor defense, and White has nothing left to try. The pawn on f4 provides the king with a safe square on f5, but it is not useful as Black's king cannot be booted off the f-file. The game is drawn.

3.堂g6 罩g1† 4.垫f5 罩f1

The fact that this move is not check is entirely irrelevant. Time to shake hands.

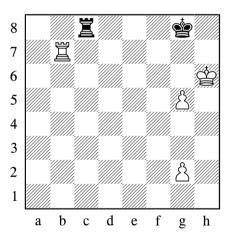
As we can see, Black could only save the game by setting up a Philidor defense, but checking from the rear lost routinely. This brings us to the first rule dealing with two doubled pawns against zero, and a rather simple one to understand.

The long and short side defense fails against doubled pawns.

This should be easy to see and is important to know and memorize. Just the knowledge is good enough, because I think the moves needed to win the game with the two pawns are very straightforward and can be worked out over the board.

Let's move on to another important exception.

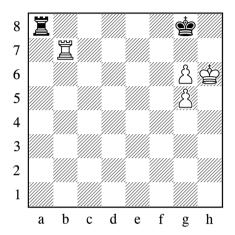
Example 3 – Pawns on g5 and g2



We have already discussed the passive defense on page 26. White would be routinely winning even without the second pawn if the whole position was shifted one file to the left, but a lone g-pawn proves insufficient for victory. In this case, however, the second g-pawn proves to be a decisive factor.

1.g6

Due to the constant threat of \(\mathbb{E}b7-b8, \text{Black's} \) rook must always remain on the back rank, but the second pawn means that White has a very convincing winning plan.



4. Zb6!

Here we see the crux of White's plan. He has placed his rook on the sixth rank to block all the checks, and then he will advance g6-g7. Without a second g-pawn, he would not be able to make any further progress, but now he can simply transfer his rook to f6 and then f8 to force a routinely winning pawn endgame.

It would be a serious error to push the pawn too soon: 4.g7? Now Black can give some checks on the sixth rank. After 4...\(\mathbb{A}a6\dagger) 5.g6\)\(\mathbb{Z}xg6\dagger!\) Black saves the game with a clever stalemate trick.

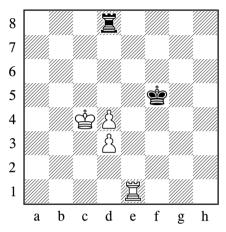
Black can resign.

Again, this leads us to an equally simple rule.

Passive defense against g-pawns fails against doubled pawns.

Let's see yet another case of a key defensive set-up failing to doubled pawns.

Example 4 – Pawns on d4 and d3



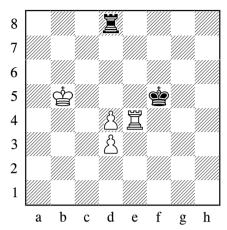
Black has set up a healthy-looking frontal defense, and since it is his move, White's pawn will not be allowed to advance to d5. But it turns out that he is lost anyway as the d3-pawn will prove to be extremely helpful.

1...罩c8† 2.垫b5 罩d8

Black plays in a similar fashion to the normal frontal defense, and we now see the key difference.

White defends his d4-pawn, and his king will easily run Black's rook out of squares in just a couple moves. The big point is that White can safely and securely keep his rook on e4, thanks to the pawn on d3. We have already seen on page 48 that the rook on e4 guarantees victory for White if Black's king is far away, and that Black needs to bring the king to the center ranks to make sure the rook cannot securely remain on such a square. As is, the black king is already on an ideal square, and it

doesn't matter in the slightest because the rook on e4 is secure.



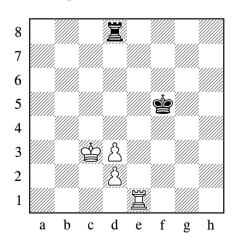
3...∲f6 4.∲c6

Now d4-d5 is a threat.

Game over. The Lucena position is coming.

Clearly, the frontal defense also saw a change to its evaluation by the presence of a second pawn. I wish I could be simplistic and say that it always fails, but unfortunately, this is not the case.

Example 5 – Pawns on d3 and d2



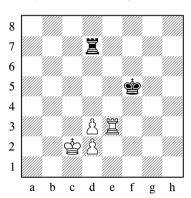
I must confess, I have never actually seen this endgame in practice. It's hard to imagine the way a game could end up with two doubled pawns on the second and third rank being the only ones remaining on the board. Still, it should be noted that here Black is saving the game because he has yet another square in between his rook and White's further advanced d-pawn.

Otherwise White would get to play d3-d4, reaching a winning position as previously seen on page 106.

Now, since White's rook cannot be harassed if it sits on a square like e3, Black's king is rather useless in the center. Conversely, Black's rook is best placed on the eighth rank. So, he should use his king to mark time.

5...**∲**f6

5...\mathbb{\mathbb{Z}}d7?! does not lose the game just yet, but it is a step in the wrong direction. Black needs four empty squares between his rook and White's d-pawn to save the game. After 6.\doc{\psi}c2

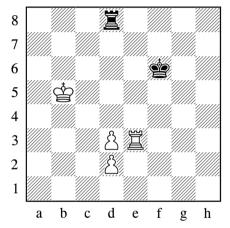


6... \model d8! would still save the game.

But 6... **E**c7†? loses, as now the rook only has three empty squares between itself and the d-pawn. 7. **b**3 **E**d7 8. **b**c3 **E**c7† 9. **b**b4 **E**d7 10. **c**5 We have seen this story before,

in the doubled pawns example just moments ago as well as in the frontal defense section on page 44. The rook will run out of squares, and d3-d4 is coming. 10...\(\mathbb{Z}c7\dagger 11.\(\dagger\dagger d6\) Black would need to play ...\(\mathbb{Z}c7\dagger d8\dagger here to save the game, and the rook does not have enough checking distance. White will get d3-d4 through, and he wins.

6. 中c2 国c8† 7. 中b3 国b8† 8. 中c3 国c8† 9. 中b4 国b8† 10. 中c5 国c8† 11. 中b5 国d8



Here we see the point. Since Black has four empty squares between his rook and the pawn, White's king coming to c6 will not be a problem since it does not threaten to advance the pawn. Instead, he must use the c5-square, and then Black will have the necessary checking distance to prevent d3-d4.

12.⊈c5

12.堂c6 堂f7! and White has nothing better than 堂c6-c5 anyway.

Finally, we see here that because Black kept his rook on the back rank, he has enough checking distance to save the game with:

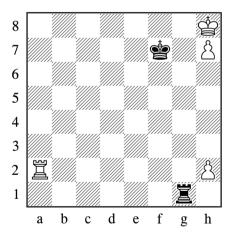
A draw can be agreed.

A frontal defense against doubled pawns on the second and third ranks is theoretically possible and deserves its own rule.

The frontal defense only works against doubled pawns with the maximum possible checking distance of four squares between the rook and the further advanced pawn. The pawn must stand on the third rank and the rook must stand on the last rank for it to hold.

Even when looking at rook pawns, an extra doubled pawn can still be very powerful. For instance, the Lucena position with a single rook pawn fails unless the opposing king is seriously misplaced, as we saw on page 19. But a second pawn can change this evaluation.

Example 6 – Pawns on h7 and h2



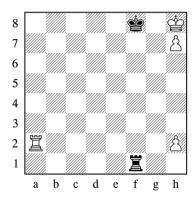
Previously, we saw a position like this one on page 17. White needed to have Black's king cut off all the way on the c-file to have any hope of winning. But the presence of a second h-pawn means that even if Black's king is as close to the pawn as possible, White can still win the game. This is because he does not need to move his rook all the way down to g8 in order to contest the g-file and free his king.

1.罩f2†!

White forces the king off the f-file. That is step one.

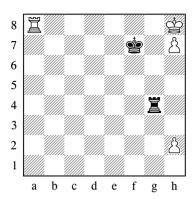
The alternatives do not win:

1.還a7†? Black's rook is unable to keep White's king cut off, but if his king is secure on the f-file, it will be a draw. Now, Black will be able to keep the king on the f7- and f8-squares for the rest of the game by transferring his rook to f1. 1... 增 8 2.還a2 It's too late for this. 2... 還f1!



The position is drawn. White's king will never escape as he cannot kick Black's king off the f-file.

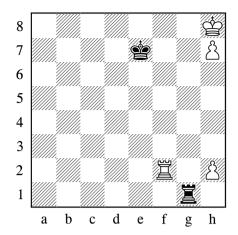
1.還a8? Normally, White would have to do something like this to contest the g-file. The problem is, both Black's king and his rook are controlling both g7 and g8. So White bringing the rook to g8 does not guarantee victory. 1...還g4 (1...還f1 is the easiest drawing move, the same as against 1.還a7?, but I am keeping ...還g1-g4 as my main line to illustrate a point.)



2.還f8†?? If White could play this move without fearing the loss of his rook, he would win the game by forcing Black's king off the f-file and then moving the rook to g8 to kick the rook away as well. But 罩a8-f8† contains the obvious problem that it is not safe and Black can take it. 2...空e7?? Of course Black should take the rook, but let's see how he loses if his king steps aside instead. 3.罩g8! 罩h4 Now Black's king and rook have both been pushed away, and White's king escapes via the g7-square. 4.堂g7罩g4† 5.堂h6 罩h4† 6.堂g6 罩g4† 7.堂f5 Game over.

1...**∲**e7

Black's king no longer controls g7 and g8. That's easy enough and could certainly be accomplished without the h2-pawn. But, with it still on the board, now White is also able to push Black's rook off the g-file without having to resort to the very slow maneuver of \(^2\)f2-a2-a8-g8, which would allow Black's king back to f7.



2.罩f3!

置g3 is coming, and Black is done for. Thanks to the second pawn giving access to g3, White is able to first give a check on the f-file, and then immediately transfer his rook to the g-file without letting Black's king return to f7 first. We saw the attempt to pull this off in the previous line with 2.罩f8†, which obviously did not work because the rook had to go to a hanging square.

It is important not to push the pawn too far:

2.h4?!

White needs to be able to transfer the rook to the g-file. One way he can do that is \(\frac{1}{2}\)-f8-g8, but this is impossible as Black's king has the f8-square under control. The other way is to transfer the rook to a square defended by the second h-pawn. This is still possible as Black's king's reach is not wide enough to control both the f8- and f5-squares, but the h4-pawn has reached its limit.

2...**¤g**3

Black can bring his king forward to stop \$\mathbb{Z}f5-g5\$ with 2...\$\dot{\phi}e6\$, but this allows White to use the back rank instead. 3.\$\mathbb{Z}f8\$ and \$\mathbb{Z}f8-g8\$ is on the way, with similar play to page 19, winning.

3.h5?

Now the position is a draw. White has two ways to contest the g-file without leaving the f-file first, namely \(\mathbb{I}f2-f8-g8\) and \(\mathbb{I}f2-f6-g6\). Black's king can comfortably sit on e7 and stop both of these plans.

After 3.\(\mathbb{I}\)f5 White wins by transferring the rook to g5.

White can't get his rook to the g-file and it is a draw.

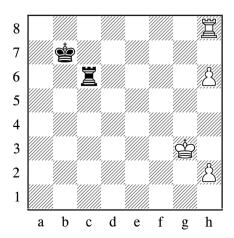
This is another very rare case, and I can't remember ever coming across it. But it feels a lot more feasible than a frontal defense against pawns on the second and third rank. Upon examination of the previous position, the next rule is obvious enough.

The Lucena position with two h-pawns is winning if the attacking side can get his rook to the f-file and his second h-pawn has not advanced past h4.

Of course, this applies in the mirrored scenario as well, only replacing every instance of "h" with "a", and "f" with "c".

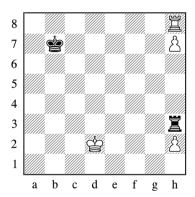
Moving on, even the Vancura defense breaks when the attacking side has a second pawn on the h-file.

Example 7 – Pawns on h6 and h2



It is somewhat surprising that an extra doubled pawn is decisive even against the Vancura defense, but in fact it is extremely simple.

1.h4!



This position perfectly highlights why the pawn needed to stand on h4.

1...\$a7 2.h7!

Now there is nothing for Black to try.

2...罩c3†

2... \Box h6 3. $\dot{\Box}$ g4 and White will easily kick the rook off the h-file with $\dot{\Box}$ g4-g5 next. The h4-pawn prevents Black's rook from reaching a square like h1.

3.∯f2

The pawn on h4 is much more effective than it was on h2. Black loses immediately as he cannot prevent the pawn from promoting.

It is impossible to definitively exhaust every single possibility, but the more positions

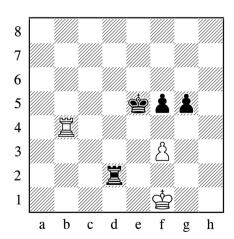
I check, the more I believe the following guideline should be followed.

The only reliable drawing set-up against two lone doubled pawns in a rook endgame is the Philidor defense. There are a couple of exceptions in the frontal defense still succeeding provided there is yet another rank of checking distance in addition to what was already required, or a Lucena position with h-pawns still being a draw if the second pawn has advanced too far. But these are very rare cases.

Rook endgames with lone doubled pawns are relatively rare, but they do happen. For example, this game was played after I wrote the rest of the chapter:

Sahil Sinha – Gregory Kaidanov

Charlotte 2021



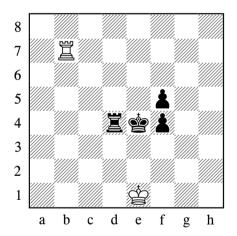
White has a relatively simple version of a 2 vs 1, and should be making an easy draw. Advancing f3-f4† was not the decisive error, but it was still a bad idea if he did not know exactly which circumstances would allow him to hold against doubled pawns. As mentioned before, the only reliable way is the Philidor defense.

85.f4†?!

This strikes me as asking for trouble.

I would rather wait with 85. = 4. As we will see later on in the 2 vs 1 on the same side section, White should be making an easy draw. His pawn did not get fixed on the second rank, and it did not get separated from his king either. 85... = 4 This is probably what Sinha was worried about, as now Black's king gets to f4, but it's no major concern. Anything holds, but I think the most convincing by far is to secure the second rank. 86. = 2 \$\frac{1}{2}\$f4 87. \$\frac{1}{2}\$f2 White shuffles = 2-b2-a2, and will give a check whenever Black's rook leaves the fourth rank. Game drawn.

85...gxf4 86.\$\dot\dot\end{a}e1 \boxdetad4 87.\$\boxdetab5 \dot\dot\delta\end{a}e4



White should be careful here. The Philidor defense is the only reliable drawing method, and we are not in time to set it up yet. If Black's king safely reaches f3, he will win.

88.罩b3?

Correct is 88. \$\div e2\$! White has other holding moves, but this would be my choice. Black could advance ...f4-f3, but this only does our dirty work for us – the whole point of the Philidor Defense is to compel this move. 88...\$\textit{\pi} a4 \textit{ 89.}\textit{\pi} b3 \text{ White has set up a Philidor.} 89...\$\textit{\pi} a2\div 90.\$\div e1 \textit{ No further progress can be made.}

88...罩d3!

Just like that, it's all over. White is unable to set up a Philidor, and Black's king reaches f3.

89. □ b7 中 f3 90. □ b7 中 g2 91. □ g7 □ g3 92. □ f7 □ e3 † 93. □ d2 □ e5 94. □ g7 † □ f2 95. □ d3 f3 96. □ d4 □ e4 † 97. □ d3 □ f1 98. □ f7 □ e5 99. □ d4 □ e1 100. □ x f5 f2 101. □ f7 □ e2 102. □ e7 † □ d2 0-1

Lone doubled pawns do not occur often in rook endgames, but luckily a little knowledge goes a long way. Once you know that the only reliable drawing setup is the Philidor defense, everything else can be worked out at the board.