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Foreword

Solve your endgame problems with Endgame Play!

Every chess player faces the problem of how to study the endgame. Three possible approaches are to ignore the endgame completely, or to read a theoretical manual like Dvoretsky’s *Endgame Manual*, or to watch videos or DVDs. In my opinion the first option is completely out of the question, as studying the endgame will repay high dividends because your understanding of the whole game and the capabilities of every single piece will improve dramatically. The knowledge gained will remain valid forever and will not become outdated like opening analysis. I recommend the other two options of course. But they are not the complete answer. They are only the first step.

It is not enough to have read a solution in a book or seen it in a video clip. You can only do it, when you can do it. For example, to mate with bishop and knight against a lone king, you must be able to master it over the board with the clock ticking, and not by guessing moves while watching a video clip.

Solving exercises is the second step on the road to endgame mastery and this is Aagaard’s approach in his excellent *Endgame Play*. The third step is to play endgames well over the board under tournament conditions. Only then are you really mastering the endgame.

So I strongly believe in training by solving exercises and Jacob Aagaard is a real master here. I often train my own students using the books from his *Grandmaster Preparation* series, and they really help on the way to becoming an International Master or hopefully even a Grandmaster.

Sometimes I challenge my students to a solving competition but one other method I use is for me to play on the weak side of the exercises, so that my students not only have to find the right solution but can also beat me afterwards, just like in a real tournament game. Again, the best way to train is to work under tournament conditions. If you do not have a grandmaster on hand, you can of course play on the winning side against a computer program – that is, if you really want to train the hard way.

Aagaard always selects many fresh and challenging examples, and in *Endgame Play* he also manages to present fascinating positions which I had not seen before – and I have seen many endgames as this has been my main occupation for years.

Endgame study has two faces – theoretical endings sorted by material, and strategical endgames sorted by motifs. Jacob Aagaard deals with both in great depth and focuses on the practical
questions. He uses pawn endings to train the calculation of long variations and visualization. He investigates minor piece endings to illustrate the capabilities of the bishop and knight and their limitations, which is very important for every phase of the royal game.

Aagaard’s treatment of opposite-coloured bishop endings, which have a very special nature almost like a new game within chess, is very deep and he also looks at positions with more pieces, where the guideline from the middlegame comes to the forefront – opposite-coloured bishops favour the attacker and, unlike pure opposite-coloured bishop endings, have no strong drawish tendencies.

Then come rook endings, endings with queens, and endings with rook and bishop to complete the discussion of theoretical endings. I want to stress the presence of many endings with rook and bishop against rook and bishop. They are very important for the practical player, but are generally underrepresented in the literature.

Regarding strategical endings, Aagaard divides the material into the following categories: schematic thinking, weaknesses, domination, do not hurry, passed pawns, pawns in the endgame, freaky aspects including zugzwang, stalemate, fortresses and attack on the king.

Here Aagaard gives a good overview and again the proof of the pudding is in the eating: you should try really hard to solve the exercises. Only in this way will you gain a deeper understanding of the real meaning of the principles and guidelines and their exceptions. The real art of the royal game is not to know the guidelines by heart and repeat them every morning three times in front of the mirror. The real art is to develop an intuitive feeling for the exceptions and to be able to calculate and visualize variations well.

Especially impressive is Aagaard’s deep insight into the nature of fortresses and the way he deals with the very important rook endgames, where it is always difficult not to be too dry and technical but also not too complicated. He strikes this balance just right and also looks at all aspects of the endgame which are relevant for the practical player. No sophisticated studies – just the sort of questions you will have to deal with over the board.

With *Endgame Play* Jacob Aagaard has again proved convincingly that he is indeed one of the best chess authors of modern times.

GM Dr Karsten Müller
Hamburg, March 2014
Black is under a lot of pressure. In the game he did not manage to find a way out. Can you do better?
Rook-and-bishop endgames have a distinctive feel to them, as the absence of queens and knights makes it unlikely that the game will be decided by mating motifs or by elaborate tactical tricks. There are, however, a lot of other things going on, which we shall explore in this chapter.

I do not have any great insights that will help you to solve the exercises, beyond “don’t spend your time looking out for knight forks!” What we are dealing with here is a collection of non-standard positions that are a lesson in themselves.

The following game provides a good illustration of some typical tactical motifs that may occur with this material balance.

**Levon Aronian – Maxime Vachier-Lagrave**

Paris/St Petersburg 2013

Black has been under pressure for all of the game, which started in one of the most fashionable variations of the Grünfeld Defence. White has managed to push his passed d-pawn quite far up the park, and Black has serious trouble stopping it. Still, it was possible to secure a draw with accurate play.

34...\textdaggerdbl}d4? 35.d7 \textdaggerdbl}d1† 36.\textdaggerdbl}f2 c4

Black is looking for a perpetual, but White has an escape route ready.

37.g3!

The king escapes from the perpetual check and White wins the game.

37...\textdaggerdbl}d2† 38.\textdaggerdbl}f3 \textdaggerdbl}d3† 39.\textdaggerdbl}g2 \textdaggerdbl}d2† 40.\textdaggerdbl}h3 \textdaggerdbl}f6 41.d8=\textdaggerdbl} \textdaggerdbl}xd8 42.\textdaggerdbl}xd8 \textdaggerdbl}xd8 1–0

How else could Black have played? 34...\textdaggerdbl}f6? would be a complete disaster on account of 35.\textdaggerdbl}a8!, when Black loses immediately.

Black could have held the game in a nice way: 34...\textdaggerdbl}e4!!

A cunning defensive idea.
35.\textdaggerdbl}f2

This is the best try, but it meets with a nice defence.
35.d7?! \textdaggerdbl}xe7! 36.d8=\textdaggerdbl} \textdaggerdbl}xa7 should be a draw, but Black's position feels more harmonious.
35.g3?! c4 36.\textdaggerdbl}g2 looks tempting, but Black can play 36...\textdaggerdbl}e6! with ideas such as ...\textdaggerdbl}b4, ...\textdaggerdbl}e5 and ...\textdaggerdbl}f6 to neutralize the d-pawn. White cannot stop them all.

35...c4 36.\textdaggerdbl}c7
Setting up the perpetual check. This time White cannot win with the g2-g3 idea, as Black can play \( \text{...b4} \) and obtain the draw.

37.\( \text{d7} \) \( \text{d2} \)

Black secures the draw. He will either deliver perpetual check, or force a draw in a different way:

38.\( \text{f3} \) \( \text{d3} \)

I would like to mention that 34...\( \text{f4} \)?! is a poor substitute. After 35.g3 Black will lose quickly unless he goes for the same defence as in the previous line: 35...\( \text{e4} \)!

36.\( \text{d4}! \)

Setting up the perpetual check. This time White cannot win with the g2-g3 idea, as Black can play \( \text{...b4} \) and obtain the draw.

37.\( \text{d7} \) \( \text{d2} \)

Black secures the draw. He will either deliver perpetual check, or force a draw in a different way:

38.\( \text{f3} \) \( \text{d3} \)

Black is just in time with this idea. He avoids an immediate defeat, but after 39.a5 \( \text{xe7} \) 40.dxe7 c3 41.\( \text{xc7} \) c2 42.\( \text{xc2} \) \( \text{xe7} \) 43.a6\( \text{+} \) White is close to winning. Only deep analysis can determine the final outcome, but no further proof is needed as to which of

Black's options on move 34 was the correct one.

This chapter does not only hold examples with rook and bishop each, but also those where one side has a rook and the other the bishop.

Shamil Arslanov – Danny de Ruiter

Groningen 2012

Black has just won the exchange and probably wanted to play safely to get past the time control before working out how to win the game. However, passive does not mean safe and after his next move, Black must have been shocked to find himself entirely lost. Rooks are big animals: they are helpless in cages, and need to feel the wind in their arrow slits.

39...\( \text{d7}? \)

A horrible move, but for us it makes the game interesting.

39...\( \text{e8} \)!

would most probably have won. White would lose immediately after 40.d7? \( \text{xe3} \)!, which is probably what Black overlooked. But after 40.\( \text{d4} \) \( \text{e2} \)\( \text{+} \), White can still fight a little bit.

40.\( \text{b6} \) \( \text{e8} \)
40...\( \text{f6} \) 41.h4 \( \text{h6} \) might look more active, but this is of little consolation after 42.g5†! hxg5 43.hxg5† \( \text{f7} \) 44.g6†! when the pawn cannot be taken and Black will soon lose.

41.\( \text{e6} \) \( \text{f7} \) 42.g5

Preventing the rook from coming to \( \text{f6} \). Black can do nothing but sit and watch White improve his position.

42...\( \text{d7} \)

42...h6 43.h4 does not change anything.

43.a4 \( \text{f7} \) 44.a5 \( \text{d7} \)

No, it is not a threefold repetition – but nice try!

45.h4 \( \text{f7} \) 46.b4 \( \text{d7} \) 47.d4 \( \text{f7} \) 48.h5 \( \text{g6} \)

48...\( \text{d7} \) 49.g6 hxg6 50.hxg6 is also hopeless, for example: 50...b5 51.\( \text{d5} \) ! \( \text{d8} \) 52.\( \text{b6†} \) \( \text{c8} \) 53.\( \text{e6} \) \( \text{b7} \) 54.\( \text{c7} \) \( \text{a7} \) 55.\( \text{c7} \) and White wins.

49.hxg6 hxg6 50.\( \text{f6} \) \( \text{d7} \) 51.\( \text{c7} \)

Black is finally in zugzwang.

51...b6 52.axb6 \( \text{b7} \)

This is the key position both players have been anticipating. Black might have hoped that his opponent had not seen how to win this position, or maybe he had not seen what was coming.

53.d7†!

White wins an important tempo by giving up his once favourite passed pawn.

53...\( \text{xd7} \) 54.\( \text{d6} \) \( \text{d8} \)

54...\( \text{h7} \) 55.\( \text{c7} \) wins.

55.\( \text{d5} \) \( \text{b7} \) 56.\( \text{e6} \) \( \text{c8} \) 57.\( \text{c7} \)

White had a slightly simpler win with 57.\( \text{c5} \)!, when after 57...\( \text{e7} \) 58.\( \text{c7} \) Black should consider resigning.

57...a5

Obviously the only move, so the trap has no chance of success.

58.\( \text{b5} \)!

58.bxa5?? \( \text{xc7†} \) is a draw.

58...\( \text{xb4} \) 59.\( \text{xb4} \) \( \text{d7} \) 60.\( \text{b5} \)!

Did White really see this on move 57, or was he just lucky?

60.\( \text{e5??} \) \( \text{xc7†} \) 61.bxc7 \( \text{xc7} \) would give Black the opposition and thus ensure a draw.
60...c8 61.c4!
Triangulation. The c5-square continues to be mined.

61...xc7†
61...d7 62.d5 is no solution either.

62.bxc7 xc7 63.c5 d7 64.d5
White wins in our favourite pawn ending.
1–0

A more extreme example of our subject is the following mad and fascinating game, which has gone in and out of the exercise folder throughout the last few years. I love it, but maybe it does not work as an exercise.

Tomas Studnicka – Lukas Cernousek
Prague 2003

31...a4? 32.d3 a3 33.d1† c6 34.d4 a2 35.cc1 b5 36.d3 a4 37.h4??
The losing move.

White could have won with: 37.c2! a3

38.b1!!, which is difficult to see, of course.
37..b3 38.h5 c2 39.h1 b1= 40.h6 a1= 41.h7 xc1 42.xc1 xc1 43.h8= d1† 0–1

In the initial position, the correct move is:
31..c6!
Protect your bishop! Actually, it makes just as much sense to put the king on d6, but since both moves lead to a draw, I have decided to include only one of them.

32.d3
The natural alternative is:
32.e2 a4 33.f4!

The passive 33.ee1? allows Black to win surprisingly easily. 33..a3 34.d3 a2 35.c2 axb1=† 36.xb1 d5 37.b3 c4 38.c2 d4→
33.\textit{\textbf{d}3}! enables White to make a draw, but the path is tricky. 33...\textit{\textbf{d}5} 34.\textit{\textbf{d}1}! (34.\textit{\textbf{e}xe5}†? \textit{\textbf{e}xe5} just wins for Black as the a-pawn advances.) 34...\textit{\textbf{xf}6} 35.\textit{\textbf{e}e8}! b3 36.\textit{\textbf{e}e}3† (36.\textit{\textbf{e}d}8†? \textit{\textbf{e}e}6 37.\textit{\textbf{e}e}8† \textit{\textbf{e}f}5 is winning for Black.) 36...\textit{\textbf{e}e}6 37.\textit{\textbf{e}e}8† Now 37...\textit{\textbf{b}7}? 38.\textit{\textbf{b}xe3} \textit{\textbf{xc}3} 39.\textit{\textbf{d}d}3 puts Black in trouble, but 37...\textit{\textbf{b}5} 38.\textit{\textbf{d}d}3 is drawing. The position is similar to the main line below; indeed, after 38...\textit{\textbf{c}2} 39.\textit{\textbf{xc}2} a transposition has occurred to move 40 in the main bolded line.

33...\textit{\textbf{xf}6}

\textbf{34.\textit{\textbf{d}d}1}!

White prepares to drive the enemy king back.

34...b3!

34...\textit{\textbf{a}3}? 35.\textit{\textbf{f}5}† \textit{\textbf{c}c}7 36.\textit{\textbf{d}d}3† \textit{\textbf{f}f}8 37.\textit{\textbf{c}c}2 a2 38.\textit{\textbf{b}b}3xa2 wins.

35.\textit{\textbf{g}4}!

The most challenging, but I also analysed: 35.\textit{\textbf{f}5}† \textit{\textbf{c}c}7 36.\textit{\textbf{d}d}5† \textit{\textbf{f}f}8

35...\textit{\textbf{a}3}!! 36.\textit{\textbf{f}f}3† \textit{\textbf{c}c}5 37.\textit{\textbf{g}5} \textit{\textbf{f}6}!

37...\textit{\textbf{f}f}5? 38.\textit{\textbf{c}c}5† \textit{\textbf{g}g}6 39.\textit{\textbf{g}g}4 c2 40.\textit{\textbf{f}f}5† \textit{\textbf{g}g}7 41.\textit{\textbf{f}f}6† \textit{\textbf{g}g}6 42.\textit{\textbf{d}d}8 and Black is mated. 38.\textit{\textbf{g}xf6} \textit{\textbf{xf}6} 39.\textit{\textbf{xe}5} -

39...\textit{\textbf{b}1}=\textit{\textbf{w}!!} 40.\textit{\textbf{x}xb}1 c2 41.\textit{\textbf{b}b}1 \textit{\textbf{b}2}

Despite being two rooks up, White has nothing better than a perpetual check.

\textbf{32.\textit{\textbf{d}d}5} 33.\textit{\textbf{e}e}2

33.\textit{\textbf{d}d}1 a4 34.\textit{\textbf{e}e}2 transposes.

33...\textit{\textbf{a}4} 34.\textit{\textbf{d}d}1! \textit{\textbf{xf}6} 35.\textit{\textbf{e}e}8! b3 36.\textit{\textbf{e}e}3†
White draws against all attempts, for example:

36...\texttt{c4} 37.\texttt{\texttt{c}8} 38.\texttt{d}3 \texttt{c2}

\textbf{39.\texttt{\texttt{c}4}! \texttt{\texttt{b}5}}

This is slightly more comfortable than drawing with the exchange less after 39...\texttt{a3} 40.\texttt{xc}2 \texttt{bxc}2 41.\texttt{xc}2 \texttt{a2} 42.\texttt{h}4 \texttt{d}4 43.\texttt{f}3 \texttt{a3} 44.\texttt{h}5 \texttt{g}7 45.\texttt{f}4 \texttt{f}5 46.\texttt{b}1 \texttt{h}6 47.\texttt{h}1 \texttt{f}8 48.\texttt{b}1, when White cannot make progress.

40.\texttt{xc}2 \texttt{a3} 41.\texttt{e}3 \texttt{a2} 42.\texttt{xb}3! \texttt{c}5 43.\texttt{xb}2 \texttt{xb}2 44.\texttt{e}4

White will hold a draw by putting his king on f5 and advancing the g-pawn. It is not possible for Black to move the king to c2 and play ...\texttt{c}1 and win, because White will have time to advance the f-pawn, move the rook to g1 and give a check on g2 with the idea to eliminate the a-pawn.

How could Black have made the right decision in the game? Essentially it is all about calculation; it is a big part of chess and often an even bigger part of the endgame. In the following exercises, deep calculation will be required as well.
1. Kitaev – Belokurov, Jaroslav 1989

2. E. Berg – Elness, Oslo 2011

3. Dzagnidze – Sargissian, Gibraltar 2012

4. B. Socko – Gajewski, Chorzow 2013


1. Kitaev – Belokurov, Jaroslavl 1989
Okay, a simple one to start. Mate in three:
1...\text{g}3†
After 1...\text{xc}7? 2.\text{h}2\text{h}3†! 3.\text{xh}3 3.\text{g}1 \text{e}3# 3...\text{g}4# 0–1

2. Emanuel Berg – Frode Elsness, Oslo 2011
33...\text{c}2! Forcing White into the pin. 33...\text{f}8 34.\text{f}4† gives White some drawing chances.
34.\text{xc}2 \text{f}8! Black is preparing ...\text{e}8, winning a piece. But not 34...\text{e}5? 35.\text{d}8†! when Black has to struggle for a draw.
35.\text{d}3 \text{e}8 36.\text{d}6 \text{xc}7 37.\text{xa}6 White struggled on, but eventually lost on move 63. 0–1

In this position White lapsed in concentration and played a really awful move.
The game went:
36.a3? \text{g}5 37.\text{e}3 \text{xe}4† 38.\text{xe}4 \text{xe}4 39.\text{xe}4 \text{g}7 40.\text{f}5 \text{h}6
The reason this endgame is far worse than the direct version is that the b3-pawn has been weakened.
White lost after:
41.\text{a}4 \text{h}4 42.\text{xe}4 \text{xe}4 38.\text{xe}4 \text{h}7 39.\text{f}5 \text{h}6 40.\text{e}6 White has no problems holding the position, for example:
40...g5 41.\text{f}5 h4 42.\text{g}xh4 gxh4 43.\text{g}4 \text{g}6 44.\text{h}xh4 \text{f}5 45.\text{h}5 \text{e}4 46.\text{g}5 \text{d}3 47.\text{f}5 \text{c}3 48.\text{e}5 \text{b}2 49.\text{d}5 \text{xa}2 50.\text{xc}5 \text{xb}3 51.\text{b}5 a4 52.c5 a3 53.c6 a2 54.c7 a1=\text{\$} 55.c8=\text{\$} With an easy draw.

4. Bartosz Socko – Grzegorz Gajewski, Chorzow 2013
Exercises with four pieces are fun, even though they usually are quite simple.
76.\text{b}8†! \text{a}7 77.\text{b}5! 1–0

58.\text{g}6! Not a difficult position, but it is always good to warm up with simple tactics.
58...\text{e}6 58...\text{g}3 59.f7† \text{h}8 60.\text{c}8 is mate on the next move.
59.\text{xc}4 Black has no defence against \text{e}8 and \text{f}4-h6, mating.
59...\text{a}6 60.\text{c}8 \text{a}7 Trying to avoid the plan.
61.\text{d}6 \text{f}7 62.\text{xf}8 Mate is near, so Black resigned. 1–0

Black has put all his chips on the two passed pawns. There is only one way to stop them and win the black rook along the way: 69.\text{e}8! 69.d8=\text{\$}? is a mistake: 69...\text{xd}8 70.\text{xd}8 h2 71.\text{h}7 g2 White will have to struggle for the draw. 69.\text{g}7 \text{a}8 70.\text{g}3 \text{d}8 would mean torture for Black, but a draw is still the likely outcome.
69...\text{xd}7 70.\text{g}8 \text{h}2 71.\text{g}7†! This was the point, of course. Instead after 71.\text{h}8?? \text{xc}7 Black wins.
71...\text{c}6 72.\text{h}7 1–0